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Alloys with random magnetic anisotropy

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Abstract. Rare-earth-based magnetic alloys are formed with local random anisotropy, which is weak or strong depending on the type of rare earth used. A continuum model is used to study the correlations, susceptibility and magnetisation as functions of the anisotropy and magnetic field.

1. Introduction

Rare-earth-based alloys have been studied extensively over the last decade. In these alloys, random, uniaxial anisotropy plays an important role in modifying the magnetic properties, which are due to the other magnetic interactions present, mainly the average and the random exchange interactions. The Harris–Plischke–Zuckerman (HPZ) (Harris *et al* 1973) model Hamiltonian

$$\mathcal{H} = -\sum_{ij} J_{ij} \boldsymbol{s}_i \cdot \boldsymbol{s}_j - D \sum_i (\hat{\boldsymbol{n}}_i \cdot \boldsymbol{s}_i)^2 - g\mu_{\mathrm{B}} \boldsymbol{H} \cdot \sum_i \boldsymbol{s}_i \tag{1}$$

is commonly used to describe these systems. Here, J_{ij} is the exchange coupling between the ith and jth spins, which has an average value J_0 and random fluctuations ΔJ, $\hat{\boldsymbol{n}}_i$ is a unit vector in the direction of the random local anisotropy axis and $\boldsymbol{H}$ is an external magnetic field.

Sellmyer and Nafis (1985) suggested a schematic phase diagram, which seems to emerge from the experimental data.

There is ample evidence (e.g. Cornelison and Sellmyer 1984, Sellmyer and O'Shea 1984) that for strong anisotropy the system is in a speromagnetic (SM) phase, where the spins are frozen in the directions of the local random anisotropy. For weak anisotropy, on the other hand, the system tends to order, ferromagnetically or antiferromagnetically, according to the type of exchange present. However, the anisotropy upsets this order to form a so-called correlated speromagnet (CSM) or correlated spin glass (CSG) (O'Shea *et al* 1983, Dieny and Barbara 1985).

From the experimental results that have been reported on weak-anisotropy alloys, mainly those with Gd, La or Eu, the following conclusions may be drawn: (i) The AC susceptibility shows a fairly sharp rise at T_c to the limiting demagnetisation value $1/N_d$, where N_d is the demagnetisation factor of the sample. (ii) Magnetisation measurements as a function of external field below T_c show the magnetisation to be completely reversible

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and the initial slope $\mathrm{d}M/\mathrm{d}H$ to be proportional to $(J/D)^4$ and much larger than the demagnetisation value. For larger fields $M(H)$ fits rather well the expression

$$M = M_0(1 - \Delta H^{-1/2}) + \chi_{\mathrm{HF}} H$$

with M_0, A and χ_{HF} (high-field susceptibility) as fitting parameters (Sellmyer and Nafis 1985).

Alloys with large D/J_0 ratio, such as is the case for alloys with the rare earths Nd, Tb, Dy or Er (Cornelison and Sellmyer 1984), have a spin-glass-like behaviour, namely, the AC susceptibility is spiked at some temperature T_{f}, where a transition to a frozen magnetisation state occurs. (Whether this is a true phase transition or not is still an open question, both theoretically and experimentally, similar to the situation in an exchange disordered spin glass.) The magnetisation $M(H)$ has a slow curved rise as H is increased.

The correlation function between spins can be inferred from neutron scattering. A squared Lorentzian seems to fit the data for the Fourier transform of the correlation function (Yoshizawa *et al* 1982, Aeppli *et al* 1984). This also received some support from theory (Feigelman and Tsodyks 1986, Chudnovsky 1987). Alben *et al* (1978) and later Chudnovsky and Serota (1983) and Chudnovsky *et al* (1986) used a continuum version of the Hamiltonian (1), writing

$$\mathcal{H} = \int \mathrm{d}^3x \, \{\tfrac{1}{2}\alpha[\nabla \boldsymbol{M}(\boldsymbol{x})]^2 - \beta[\boldsymbol{M}(\boldsymbol{x}) \cdot \hat{\boldsymbol{n}}(\boldsymbol{x})] - \boldsymbol{M}(\boldsymbol{x}) \cdot \boldsymbol{H}\}. \tag{2}$$

Here α and β are related to the exchange and anisotropy parameters J and D of (1), by

$$\alpha = \frac{Ja^2}{M_0^2} z \frac{N}{V} \tag{3}$$

and

$$\beta = \frac{D}{M_0^2} \frac{n}{V} \tag{4}$$

where M_0 is the saturation magnetisation, z is the number of nearest neighbours to which each spin is coupled, N is the number of spins included in the sum (1), a is the magnetic lattice spacing and V is the volume of integration of (2).

We assume that only the direction of $\boldsymbol{M}(\boldsymbol{x})$ varies in space, while its magnitude is constant and equals the saturation value. Denoting the polar angles of $\boldsymbol{M}(\boldsymbol{x})$ and $\hat{\boldsymbol{n}}(\boldsymbol{x})$ by $\Theta(\boldsymbol{x})$ and $\Theta_{\mathrm{A}}(\boldsymbol{x})$ and assuming that $\boldsymbol{M}(\boldsymbol{x})$ follows $\hat{\boldsymbol{n}}(\boldsymbol{x})$, in the azimuthal direction, $\mathcal{H}$ becomes

$$\mathcal{H} = \frac{\alpha M_0^2}{a^2} \int \mathrm{d}^3x \, \tfrac{1}{2}\{a^2(\nabla\Theta)^2 - h_{\mathrm{A}} \cos^2[\Theta(\boldsymbol{x}) - \Theta_{\mathrm{A}}(\boldsymbol{x})] - h \cos\Theta(\boldsymbol{x})\} \tag{5}$$

where

$$h_{\mathrm{A}} = \frac{\beta a^2}{\alpha} = \frac{D}{Jz} \tag{6}$$

and

$$h = \frac{HM_0}{zJ} \frac{V}{N} = \frac{H}{H_{\mathrm{ex}}} \tag{7}$$

measure the strength of the anisotropy and of the external field relative to the exchange field, H_{ex}. This Hamiltonian will be our starting point in the following sections.

2. The basic equation

The properties of a magnetic system that depends on random variables, such as the local anisotropy directions, must be determined statistically. For this purpose let us define the partition function Z by the functional integral

$$Z = \int \mathscr{D}\Theta(\cdot) \exp(-\mathscr{H}[\Theta]/T). \tag{8}$$

Defining the equilibrium angles $\Theta(x)$ by

$$\delta\mathscr{H}/\delta\Theta(x) = 0 \tag{9}$$

leads to the equation

$$a^2\nabla^2\Theta(\boldsymbol{x}) = h_A \sin\{2[\Theta(\boldsymbol{x}) - \Theta_A(\boldsymbol{x})]\} + h \sin\Theta(\boldsymbol{x}) \tag{10}$$

which the equilibrium angles must satisfy. Hence the partition function is

$$Z = \tilde{Z} \exp(-\mathscr{H}_{eq}[\Theta]/T) \tag{11}$$

where

$$\tilde{Z} = \int \mathscr{D}\tilde{\Theta}(\cdot) \exp(-\mathscr{H}_2[\tilde{\Theta}]/T) \tag{12}$$

and $\mathscr{H}_{eq}$ is the Hamiltonian (5), where the angles $\Theta(x)$ assume their equilibrium values of equation (10). In equation (12), $\tilde{\Theta}$ are the fluctuations around the equilibrium and $\mathscr{H}_2[\tilde{\Theta}]$ is given by

$$\mathscr{H}_2[\tilde{\Theta}] = \frac{\alpha M_0^2}{a^2} [\![\tfrac{1}{2}a^2(\nabla\tilde{\Theta})^2 + \{h_A \cos[2(\Theta(\boldsymbol{x}) - \Theta_A(\boldsymbol{x}))] + \tfrac{1}{2}h\cos\Theta(\boldsymbol{x})\}\tilde{\Theta}^2(\boldsymbol{x})]\!] \tag{13}$$

where again $\Theta(\boldsymbol{x})$ satisfies equation (10).

The random local anisotropy direction, $\Theta_A(\boldsymbol{x})$, is assumed to be characterised by an average $\langle\sin\Theta_A(\boldsymbol{x})\rangle = 0$ and a Gaussian distribution, which leads to

$$\begin{aligned}\langle\sin\Theta_A(\boldsymbol{x})\sin\Theta_A(\boldsymbol{y})\rangle &= \langle\cos\Theta_A(\boldsymbol{x})\cos\Theta_A(\boldsymbol{y})\rangle \\ &= \tfrac{1}{2}\exp\{-\langle[\Theta_A(\boldsymbol{x}) - \Theta_A(\boldsymbol{y})]^2\rangle/2\}.\end{aligned} \tag{14}$$

It will be shown later that it is consistent to assume that the random directions are correlated over a given correlation length R_A, such that

$$\tfrac{1}{2}\langle[\Theta_A(\boldsymbol{x}) - \Theta_A(\boldsymbol{y})]^2\rangle = |\boldsymbol{x} - \boldsymbol{y}|/R_A \tag{15}$$

and the average $\langle\ \rangle$ is over the distribution of the random anisotropy directions.

As a result of the randomness in the anisotropy, the local magnetisation direction $\Theta(\boldsymbol{x})$ will also be random. The statistical properties of $\Theta(\boldsymbol{x})$ are, in principle, determined by equation (10). This direct route, e.g. by a derivation of a Fokker–Planck equation for the distribution function $W\{\Theta(\boldsymbol{x})\}$, is beyond the scope of this paper and will be discussed elsewhere. Here we shall adopt an approximate, but simple, approach to define average quantities, such as correlation function or magnetisation. Different approximations are, however, appropriate for weak or strong anisotropies or for strong magnetic field. We shall therefore discuss each of these cases separately in the following sections.

3. Weak anisotropy

We use the smallness of the source term in equation (10), since both the anisotropy and the external magnetic field are assumed to be small, to write the formal solution of equation (10) as

$$\Theta(\boldsymbol{x}) = \frac{1}{a^2}\int \mathrm{d}^3x'\, G(\boldsymbol{x},\boldsymbol{x}')[\![h_{\mathrm{A}} \sin\{2[\Theta(\boldsymbol{x}') - \Theta_{\mathrm{A}}(\boldsymbol{x})]\} + h \sin\Theta(\boldsymbol{x})]\!] \tag{16}$$

where $G(\boldsymbol{x}, \boldsymbol{y})$ is the Green function

$$G(\boldsymbol{x},\boldsymbol{y}) = -(1/4\pi)/|\boldsymbol{x}-\boldsymbol{y}|. \tag{17}$$

3.1. *Correlation function*

Let us now express $[\Theta(\boldsymbol{x}) - \Theta(\boldsymbol{y})]^2$ using equation (16) and then take the average of this expression. In taking the average of a functional of Θ and of Θ_{A} we shall use the weak-anisotropy approximation, which in this context means that

$$\langle F\{\Theta, \Theta_{\mathrm{A}}\}\rangle = \langle\langle F\{\Theta, \Theta_{\mathrm{A}}\}\rangle_{\mathrm{A}}\rangle_{\mathrm{M}} \tag{18}$$

where $\langle\ \rangle_{\mathrm{A}}$ means averaging over the random anisotropy directions and $\langle\ \rangle_{\mathrm{M}}$ means averaging over the random direction of the magnetisation. In performing the first average the angles Θ are considered non-random variables.

We assume that $\Theta(\boldsymbol{x})$ is also a Gaussian random variable and that, similar to $\Theta_{\mathrm{A}}(\boldsymbol{x})$, it has a correlation length R_{M}, which obeys

$$\langle \sin\Theta(\boldsymbol{x}) \sin\Theta(\boldsymbol{y})\rangle = \tfrac{1}{2}\exp\{-\tfrac{1}{2}\langle[\Theta(\boldsymbol{x}) - \Theta(\boldsymbol{y})]^2\rangle\} = \tfrac{1}{2}\exp(-|\boldsymbol{x}-\boldsymbol{y}|/R_{\mathrm{M}}) \tag{19}$$

and using equations (16) and (18) for the average $\langle[\Theta(\boldsymbol{x}) - \Theta(\boldsymbol{y})]^2\rangle$, we find a self-consistent expression for R_{M}.

In the weak-anisotropy limit $R_{\mathrm{M}} \gg R_{\mathrm{A}}$, which is the justification for the procedure suggested by equation (18). This procedure, unlike the one used by Chudnovsky *et al* (1986), leads to a self-consistent dependence of $R_{\mathrm{M}}/R_{\mathrm{A}}$ on the magnetic field, namely

$$R_{\mathrm{A}}/R_{\mathrm{M}} \equiv \gamma_{\mathrm{M}} = \tfrac{1}{2}[\tilde{h}_{\mathrm{A}}^2(1+\gamma_{\mathrm{M}})^{-3} + \tilde{h}^2\gamma_{\mathrm{M}}^{-3}] \tag{20}$$

where

$$\tilde{h}_{\mathrm{A}} = \tfrac{1}{8}(R_{\mathrm{A}}/a)^2 h_{\mathrm{A}} \tag{21}$$

$$\tilde{h} = (R_{\mathrm{A}}/a)^2 h. \tag{22}$$

In the above, the following approximation was used

$$\frac{1}{4\pi}\int \mathrm{d}^3\xi(|\boldsymbol{\xi} + \boldsymbol{\Delta}| - \xi)\,\mathrm{e}^{-\xi/R} \simeq 2|\boldsymbol{\Delta}|R^3. \tag{23}$$

In the zero-field limit

$$\gamma_{\mathrm{M}}(0) = \tfrac{1}{2}\tilde{h}_{\mathrm{A}}^2(1+\gamma_{\mathrm{M}})^{-3} \simeq \tfrac{1}{2}\tilde{h}_{\mathrm{A}}^2(1 - \tfrac{3}{2}\tilde{h}_{\mathrm{A}}^2 + \dots). \tag{24}$$

For $h \neq 0$ but $h \ll \frac{1}{8}\tilde{h}_{\mathrm{A}}^4$

$$\gamma_{\mathrm{M}}(h) \simeq \gamma_{\mathrm{M}}(0)(1 + 8\tilde{h}^2/\tilde{h}_{\mathrm{A}}^4) \tag{25}$$

and for $h \gg \frac{1}{8}\tilde{h}_{\mathrm{A}}^4$

$$\gamma_{\mathrm{M}}(h) \simeq (\tilde{h}/\sqrt{2})^{1/2} + \tfrac{1}{8}\tilde{h}_{\mathrm{A}}^2. \tag{26}$$

3.2. *Susceptibility*

The average magnetisation in the direction of the external field can be written as

$$M_z = \frac{M_0}{V}\int d^3x \cos\Theta(\boldsymbol{x}) \tag{27}$$

and the susceptibility is given by

$$\chi = \left\langle\frac{\partial M_z}{\partial H}\right\rangle = -\frac{M_0}{H_{ex}}\frac{1}{V}\int d^3x\left\langle \sin\Theta(\boldsymbol{x})\,\frac{\partial\Theta(\boldsymbol{x})}{\partial h}\right\rangle. \tag{28}$$

We use $\Theta(\boldsymbol{x})$ from equation (16) and the averaging procedure as in (18) and find

$$\chi = \frac{M_0}{H_{ex}}\frac{1}{2}\left(1 + \tfrac{1}{2}\tilde{h}\frac{\partial}{\partial\tilde{h}}\right)\gamma_M^{-2}. \tag{29}$$

Using equations (25) and (26) we find

$$\chi = \chi_0(1 - B\tilde{h}^2) \tag{30}$$

where

$$\chi_0 = \frac{1}{2}\frac{M_0}{\tilde{H}_{ex}}\frac{1}{\gamma_M(0)}\frac{\partial}{\partial h}\left(\frac{\tilde{h}}{\gamma_M(h)}\right)\bigg|_{h=0} = \frac{M_0}{H_{ex}}\frac{2}{\tilde{h}_A^4} \tag{31}$$

and

$$B = 32/\tilde{h}_A^4. \tag{32}$$

With

$$\tilde{h}_A = \frac{1}{8}\left(\frac{R_A}{a}\right)^2\left(\frac{D}{ZJ}\right)$$

as given by equations (6) and (21), the initial susceptibility can be seen to be proportional to $(J/D)^4$, as previously found by Aharony and Pytte (1980, 1983) and by Chudnovsky and Serota (1983) and Chudnovsky *et al* (1986), and is hence very large for weak anisotropy. This means that the magnetisation tends to saturate for extremely small fields h, of the order of $(\tilde{h}_A^4/8)$, so that, for weak anisotropy, even weak external fields should be considered strong.

Barbara and Dieny (1985) analysed χ_0, the initial slope of the magnetisation, in samples of $Dy_xGd_{1-x}Ni$ for $1 \leqslant x \leqslant 0.25$. In these samples D/J increases with x. They found $\chi_0 \propto (J/D)^{3.8\pm0.2}$. Sellmyer and Nafis (1985) find that the magnetisation curve for $Gd_{72}Fe_{10}Ga_{18}$ has a very large initial slope—the magnetisation nearly saturates ($M/M_0 \sim 0.9$) for fields of ~ 1 kOe and then complete saturation is not reached until $H \sim 100$ kOe.

It is therefore necessary to check now the strong-field case.

4. Strong field

As seen in the previous section the magnetisation approaches saturation for fields in excess of $(\tilde{h}_A^4/8)$; therefore a field of this magnitude nearly aligns the magnetic moments, i.e. the angles $\Theta(\boldsymbol{x})$ anywhere are small (compared with any other relevant angle such as $\Theta_A(x)$). Notice that even though this field is strong, in the sense discussed above, it still can and in most cases probably will be small compared with the exchange field, i.e. $\tilde{h} \ll 1$.

Equation (10) is now approximated by dropping $\Theta(x)$ compared with $\Theta_A(x)$ and by replacing $\sin\Theta(x)$ by $\Theta(x)$ in the last term on the RHS of equation (10). The solution for $\Theta(x)$ is

$$\Theta(x) = -\frac{h_A}{a^2}\int d^3y\, G_h(x-y)\sin[2\Theta_A(y)] \tag{33}$$

where the Green function $G_h(x-y)$ is given by

$$G_h(x-y) = -\frac{1}{4\pi|x-y|}\exp\left(-\frac{|x-y|}{R_h}\right) \tag{34}$$

where

$$R_h = a/h^{1/2}. \tag{35}$$

4.1. Correlation function

Using the above solution for $\Theta(x)$ we can express the average $\langle[\Theta(x)-\Theta(x)]^2\rangle$ and see how this depends on $|x-y|$. Making use of the identity

$$\int d^3y\, G_h(y)G_h(y+z) = (1/8\pi)R_h\exp(-z/R_h) \tag{36}$$

we find that for $|x-y|$ smaller than either R_h or R_A

$$\langle[\Theta(x)-\Theta(y)]^2\rangle = (x-y)^2/R_F^2. \tag{37}$$

This means that the correlation function $\langle\sin\Theta(x)\sin\Theta(y)\rangle$ is a Gaussian with a width R_F, where

$$R_F/R_A = \begin{cases}\sqrt{(3/2)}\tilde{h}_A^{-1}(1+3/8\tilde{h}^{1/2}) & \text{for } \tilde{h}^{1/2} \ll 4\\ \frac{1}{8}\sqrt{(3/2)}\tilde{h}_A^{-1}\tilde{h}^{3/4}(1+6\tilde{h}^{-1/2}) & \text{for } \tilde{h}^{1/2} \gg 4.\end{cases} \tag{38}$$

For $|x-y|$ large compared with both R_h and R_A we find

$$\langle[\Theta(x)-\Theta(y)]^2\rangle = \begin{cases}\tilde{h}_A^2/\tilde{h}^{1/2}[1-\exp(-|x-y|/R_h)] & \text{for } \tilde{h}^{1/2} \ll 4\\ 64\tilde{h}_A^2/h^e[1-\exp(-4|x-y|/R_A)] & \text{for } \tilde{h}^{1/2} \gg 4.\end{cases} \tag{39}$$

In this case the correlation function $\langle\sin\Theta(x)\sin\Theta(y)\rangle$ becomes a constant as $|x-y|$ increases.

4.2. Susceptibility

As seen before the susceptibility is given by

$$\chi = -\frac{M_0}{H_{\mathrm{ex}}}\frac{1}{V}\int \mathrm{d}^3x\left\langle \sin\Theta(\boldsymbol{x})\frac{\partial\Theta(\boldsymbol{x})}{\partial h}\right\rangle \tag{40}$$

which can be approximated, in the present strong-field case, by

$$\chi = -\frac{M_0}{H_{\mathrm{ex}}}\frac{1}{V}\frac{\partial}{\partial h}\int \mathrm{d}^3x\langle\Theta^2(\boldsymbol{x})\rangle. \tag{41}$$

For $\Theta(\boldsymbol{x})$ we use equation (33) and find

$$\langle\Theta^2(\boldsymbol{x})\rangle = 32\tilde{h}_{\mathrm{A}}^2\tilde{h}_{\mathrm{A}}^2\tilde{h}^{-1/2}/(4+\tilde{h}^{1/2})^3. \tag{42}$$

Hence

$$\chi = \begin{cases} \dfrac{1}{8}\dfrac{M_0}{H_{\mathrm{ex}}}\left(\dfrac{R_{\mathrm{A}}}{a}\right)^2 \tilde{h}_{\mathrm{A}}^2\tilde{h}^{-3/2} & \text{for } \tilde{h}^{1/2} \ll 4 \\ 32\dfrac{M_0}{H_{\mathrm{ex}}}\left(\dfrac{R_{\mathrm{A}}}{a}\right)^2 \tilde{h}_{\mathrm{A}}^2\tilde{h}^{-3} & \text{for } \tilde{h}^{1/2} \gg 4. \end{cases} \tag{43}$$

The deviation M of the magnetisation from saturation can be calculated as follows:

$$M(h) = H_{\mathrm{ex}}\left(\frac{R_{\mathrm{A}}}{a}\right)^2\int_0^{\tilde{h}}\chi(\tilde{h})\,\mathrm{d}\tilde{h} = H_{\mathrm{ex}}\left(\frac{R_{\mathrm{A}}}{a}\right)^2\left(\int_0^{\infty}\chi(\tilde{h})\,\mathrm{d}\tilde{h} - \int_{\tilde{h}}^{\infty}\chi(\tilde{h})\,\mathrm{d}\tilde{h}\right) = M - \delta M \tag{44}$$

whence

$$\delta M/M_0 = \begin{cases} \frac{1}{4}\tilde{h}_{\mathrm{A}}^2\tilde{h}^{-1/2} & \text{for } \tilde{h}^{1/2} \ll 4 \\ 16\tilde{h}_{\mathrm{A}}^2\tilde{h}^{-2} & \text{for } \tilde{h}^{1/2} \gg 4. \end{cases} \tag{45}$$

For weak fields equation (45) is found to fit Sellmyer's data (Sellmyer and Nafis 1985) for $Gd_{72}Fe_{10}Ga_{18}$ rather well, as seen in figure 1. The parameters chosen to fit the data are $M_0 = 220$ emu g^{-1}, $\tilde{h}_{\mathrm{A}}^2 = 0.4$, $a/R_{\mathrm{A}} = 0.25$ and $H_{\mathrm{ex}} = 8$ kOe. For fields larger than 20 kOe we were not able to fit the data with a single term as in equation (45).

5. Strong anisotropy

The case of strong anisotropy occurs when either D is large or the random anisotropy correlation length R_{A} is long, since the relevant parameter is $\tilde{h}_{\mathrm{A}} \propto (R_{\mathrm{A}}/a)^2 D$. In this case it is reasonable to assume that $|\Theta - \Theta_{\mathrm{A}}|$ is small, such that

$$\sin\{2[\Theta(\boldsymbol{x}) - \Theta_{\mathrm{A}}(\boldsymbol{x})]\} = 2[\Theta(\boldsymbol{x}) - \Theta_{\mathrm{A}}(\boldsymbol{x})]. \tag{46}$$

The solution to equation (10) is

$$\Theta(\boldsymbol{x}) = \frac{1}{a^2}\int \mathrm{d}^3x'\, G_{\mathrm{A}}(\boldsymbol{x}-\boldsymbol{x}')[-2h_{\mathrm{A}}\Theta_{\mathrm{A}}(\boldsymbol{x}) + h\sin\Theta(\boldsymbol{x})] \tag{47}$$

where

$$G_{\mathrm{A}}(\boldsymbol{x}-\boldsymbol{x}') = -\frac{1}{4\pi|\boldsymbol{x}-\boldsymbol{x}'|}\exp\left(-\lambda_{\mathrm{A}}\frac{|\boldsymbol{x}-\boldsymbol{x}'|}{a}\right) \tag{48}$$

with $\lambda_{\mathrm{A}} = \surd(2h_{\mathrm{A}})$.

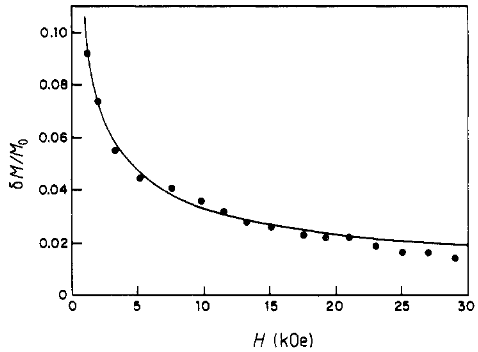

Figure 1. The relative deviation from saturation of the magnetisation for weak anisotropy. The full curve is the single-parameter curve of equation (45) chosen to fit the data (dots) of Sellmyer and Nafis (1985) for $Gd_{72}Fe_{10}Ga_{18}$.

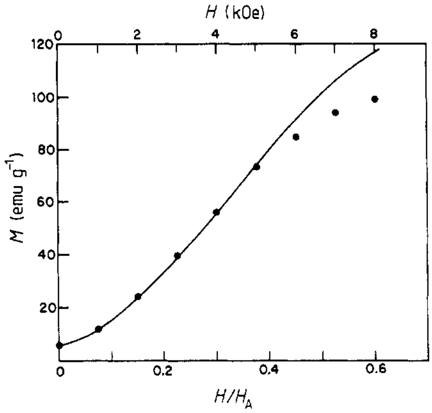

Figure 2. The magnetisation curve for strong anisotropy from equation (60). The three parameters were chosen to fit data by Cornelison and Sellmyer (1984) shown by the dots.

To conform with the previous definition of the statistics of $\Theta_A(x)$ and the anisotropy correlation length R_A, as in equation (15), we assume here that

$$\langle\Theta_A(x)\Theta_A(x')\rangle - \langle\Theta_A^2(x)\rangle = -|x - x'|/R_A. \tag{49}$$

5.1. Correlation length

The magnetic correlation length is found from equation (19), with

$$\langle\sin\Theta(x)\Theta_A(x')\rangle = \langle\Theta(x)\Theta_A(x')\rangle\exp(-\langle\Theta^2\rangle/2) = g(x - x')\exp(-\langle\Theta^2\rangle/2) \tag{50}$$

where

$$g(z) = -2h_A\frac{1}{a^2}\int d^3z'\, G_\lambda(z - z')\left(\langle\Theta_A^2\rangle - \frac{|z - z'|}{R_A}\right) \tag{51}$$

and

$$G_\lambda(z - z') = -\frac{1}{4\pi|z - z'|}\exp\left(-\lambda\frac{|z - z'|}{a}\right) \tag{52}$$

with

$$\lambda = (2h_A + h\,e^{-\langle\Theta^2\rangle/2})^{1/2}. \tag{53}$$

Substituting this into equation (18) and using (17) we find for the ferromagnetic correlation length

$$\gamma_M = R_A/R_M = \left(1 - \frac{2\tilde{h}\,e^{-\langle\Theta^2\rangle/2}}{2\tilde{h}_A + \tilde{h}\,e^{-\langle\Theta^2\rangle/2}} + \frac{1}{2}\frac{\tilde{h}^2/4\tilde{h}_A^{1/2}}{(\gamma_M + 4\tilde{h}_A^{1/2})^3}\frac{1 - e^{\Delta/R_M}}{\Delta/4a}\right). \tag{54}$$

Quite obviously, for $h = 0$, $\gamma_M = 1$, i.e. the magnetic moments follow the local anisotropy direction perfectly. As the magnetic field is increased, it prevents the magnetic moments from following the anisotropy and hence R_M decreases.

5.2. *Susceptibility*

The expression for the susceptibility, equation (40), now becomes

$$\chi = -\frac{1}{2}\frac{M_0}{H_{ex}}\left(1 + \tfrac{1}{2}h\frac{\partial}{\partial h}\right)\frac{1}{a^2}\int d^3x\, G_A(x)\langle\sin\Theta(x)\sin\Theta(0)\rangle. \tag{55}$$

Expressing this in powers of $q = h/h_A$ we find

$$\chi = \chi_0(1 + Aq - Bq^2 + \ldots) \tag{56}$$

where

$$\chi_0 = \frac{M_0}{4h_A H_{ex}} \tag{57}$$

$$A = \tfrac{9}{4}(a/R_A)h_A^{-1/2}\, e^{\langle\Theta^2\rangle/2} \tag{58}$$

and

$$B = 2(a/R_A)h_A^{-1/2}(1 + \tfrac{1}{8}\, e^{-\langle\Theta^2\rangle/2}). \tag{59}$$

This indicates that the magnetisation has an initial slope inversely proportional to h_A, and hence small; it increases as a quadratic of the field and then levels off as the negative quadratic term in χ becomes important.

There are some experimental indications for such a behaviour in Er- and Tb-based rare-earth alloys by Cornelison and Sellmyer (1984). Their magnetisation data for $(Er_{80}Ga_{20})_{80}Fe_{20}$ are compared to the magnetisation curve based on equation (56), namely

$$M = M_r + \chi_0 H_A(q + \tfrac{1}{2}Aq^2 - \tfrac{1}{3}Bq^3) \tag{60}$$

and are shown in figure 2. The parameters that fit the weak-field data, with a saturation magnetisation value $M_0 = 144$ emu g^{-1}, are $M_r = 6$ emu g^{-1}, $\chi_0 = 2.7$ emu g^{-1} kOe^{-1}, $H_A = 13.3$ kOe, $A = 35.16$ and $B = 53.34$.

6. Conclusions

A continuum HPZ model of alloys with random anisotropy and with external magnetic field yields self-consistently a smooth transition from very weak to intermediate and strong fields with no indication of different phases, correlated speromagnet (CSM) and ferromagnet with wandering axis (FWA), as suggested by Chudnovsky *et al* (1986). Strong anisotropy has not been treated before, except in the infinite limit as was done by Feigelman and Tsodyks (1986). The magnetisation as a function of field shows an alloy that is harder to magnetise. The correlation length first increases and then decreases with magnetic field, as the magnetic field prevents the magnetisation from freely following the local anisotropy.

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